# P.E.S. COLLEGE OF ENGINEERING: MANDYA-571 401

(An Autonomous Institution under VTU, Belgaum)

## DEPARTMENT OF MATHEMATICS

**THIRD SEMESTER B.E. – ACADEMIC YEAR: 2009-10**

**Course with Code: Engineering Mathematics-III(P08MA31)**

**(Common to all Branches)**

**Model Question Paper**

**Credits : 4-0-0** **Time: 03 Hours** **Max.Marks:100**

**NOTE:- Answer any FIVE full questions choosing at least TWO questions from each part.**

## PART - A

1. a) Obtain the Fourier series of the function $x^2$ in $-\pi \le x \le \pi$ and hence deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots = \frac{\pi^2}{12}$$ {06}

b) Find the Fourier series of the periodic function defined by $f(x) = 2x - x^2$ in the interval $0 < x < 3$ {07}

c) Find the Fourier series of the function y upto second harmonics using the following data {07}

| $x$ | 0 | $\pi/3$ | $2\pi/3$ | $\pi$ | $4\pi/3$ | $5\pi/3$ | $2\pi$ |
|---|---|---|---|---|---|---|---|
| $y$ | 0.8 | 0.6 | 0.4 | 0.7 | 0.9 | 1.1 | 0.8 |

2. a) Find a half range cosine series for $f(x) = (x-1)^2$ , $0 \le x \le 1$ {06}

b) Find the complex Fourier transform of the function f(x) = $\begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$

hence find the value of $\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\, dx$ {07}

c) Find the Fourier sine transform of f(x) = $e^{-|x|}$ and hence evaluate

$\int_0^\infty \frac{x \sin mx}{1+x^2} dx, m > 0$ {07}

3. a) Given that

| $x$ | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 |
|---|---|---|---|---|---|---|---|
| $y$ | 7.989 | 8.403 | 8.781 | 9.129 | 9.451 | 9.750 | 10.031 |

Find $\left(\frac{dy}{dx}\right)$ and $\left(\frac{d^{2y}}{dx^2}\right)$ at x = 1.1 {06}

b) The table gives the distances in nautical miles of the visible horizon for the given heights in feet above the earth's surface

| $x =$ height | 100 | 150 | 200 | 250 | 300 | 350 | 400 |
|---|---|---|---|---|---|---|---|
| y = distance | 10.63 | 13.03 | 15.04 | 16.81 | 18.42 | 19.90 | 21.27 |

Find the values of y when $(i)\ x = 110\ ft$ $(ii)\ x = 410\ ft$ $\{07\}$

**Contd…2**



c) Use Lagrange's interpolation formula to fit a polynomial for the following data:

| $x$ | 1 | 2 | 4 | 5 |
|---|---|---|---|---|
| $y = f(x)$ | $-12$ | 0 | 6 | 12 |

Hence find the value of $f(3)$ $\{07\}$

4 a) Find the approximate value of $\int_0^{\pi\backslash 2} \sqrt{\cos\theta}\, d\theta$ by Simpson's $\left(\frac{1}{3}\right)^{rd}$ rd rule by dividing $\left[0, \frac{\pi}{2}\right]$ into six equal part. $\{06\}$

b) Use Regula Falsi method to obtain a real root of the equation $x^5 - x^4 - x^3 - 1 = 0$, correct to three decimal places. $\{07\}$

c) Derive the Newton-Raphson iterative formulato find the real root of the equation $x \log_{10} x = 1.2$ and hence find the root correct to four decimal places. $\{07\}$

## PART – B

5 a) Employ Taylor's series method to obtain approximate value of $y$ at $x = 0.1$, given that $dy/dx = 2y + 3e^x$, $y(0) = 0$ $\{06\}$

b) Using Runge - Kutta method of fourth order to find $y(0.2)$ for the equation $(x + y)\frac{dy}{dx} = 1,\ y(0) = 1$ taking $h = 0.1$ $\{07\}$

c) Use Milne's predictor-corrector method to find $y(0.4)$ given that $dy/dx = x + y$ and $y(0) = 1.0$, $y(0.1) = 1.1034$ $y(0.2) = 1.2428$ $y(0.3) = 1.3997$ $\{07\}$

6. a) Find the Z-transforms of (i) $n^2$ (ii) $\cos n\theta$. $\{06\}$

b) Find the inverse Z-transform of $\dfrac{5z^2 + 2}{(3z - 1)(3z + 2)}$ $\{07\}$

c) Using Z-transforms, solve $y_{n+2} - 2y_{n+1} + y_n = 2^n$, $y_0 = 2$, $y_1 = 4$ $\{07\}$

7. a) Form the partial differential equation by eliminating arbitrary functions from $f(x^2 + 2yz, y^2 + 2zx) = 0$ $\{06\}$

b) Solve: $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. $\{07\}$

c) Solve by the method of separation of variables: $y^3 \dfrac{\partial u}{\partial x} + x^2 \dfrac{\partial u}{\partial y} = 0$, given, $u = 2e^{-x^3}$ when $y = 0$. $\{07\}$

8. a) Obtain D'Alembert's solution of wave equation. $\{06\}$

b) Find the various possible solutions of the two dimensional Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ by the method of separation of variables. $\{07\}$

c) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ given that $u(1,t) = 0,$ $\frac{\partial u}{\partial t} = 0$ when $t = 0$ and $u(x,0) = u_0 \sin(\pi x / l).$ $\{07\}$

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## DEPARTMENT OF MATHEMATICS

### FOURTH SEMESTER B.E. – ACADEMIC YEAR: 2009-10

*Course with Code: Engineering Mathematics-IV(P08MA41)*

## *Model Question Paper*

**Credits : 4-0-0** **Time: 03 Hours** **Max.Marks:100**

**NOTE:- Answer any FIVE full questions choosing at least TWO questions from each part.**

**PART – A**

1. a) Define an analytic function and obtain Cauchy-Riemann equations in the polar form. $\{06 Marks\}$

   b) If $\phi + i\psi$ represents the complex potential of an electrostatic field where $\psi = (x^2 - y^2) + \dfrac{x}{x^2 + y^2}$, find $\phi$. Also, find the complex potential as a function of the complex variable $z = x + iy$. $\{07 Marks\}$

   c) Find the bilinear transformation which maps the points $z = 1, i, -1$ into $w = 2, i, -2$. Also, find the invariant points of the transformation. $\{07 Marks\}$

2. a) State and prove Cauchy's integral formula $\{06 Marks\}$

   b) Expand $f(z) = \dfrac{z+1}{(z+2)(z+3)}$ in terms of Laurent's series valid in the regions $(i)\ |Z| > 3 \quad (ii)\ 2 < |Z| < 3$ $\{07 Marks\}$

   c) State Cauchy's residue theorem. Using the same evaluate $\int_C \dfrac{e^{2z}}{(z+1)(z-2)} dz$, where C is the circle $|Z| = 3$. $\{07 Marks\}$

3. a) Solve in series the differential equation $\dfrac{d^2y}{dx^2} + x^2 y = 0$. $\{06 Marks\}$

   b) Solve Bessel's differential equation leading to $J_n(x)$. $\{07 Marks\}$

   c) State Rodrigue's formula. Express $4x^3 - 2x^2 - 3x + 8$ in terms of Legendre's polynomials. $\{07 Marks\}$

4. a) Define (i) skewness and (ii) kurtosis of a frequency distribution $\{x_i f_i\}_{i=1,2,\ldots..n}$. The first four moments about an arbitrary value "5" of a frequency distribution $\{x_i f_i\}_{i=1,2,\ldots..n}$ are 2, 20, 40 and 50. Find the skewness and kurtosis, based on moments. $\{06 Marks\}$

b) Fit a best fitting parabola $y = a + bx + cx^2$ for the data

| x | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| y | 3.07 | 12.85 | 31.47 | 57.38 | 91.29 |

by the method of least squares. {07*Marks*}

c) The following data gives the age of husband ($x$) and the age of wife ($y$) in years. Calculate the coefficient of correlation and lines of regression

| x | 36 | 23 | 27 | 28 | 28 | 29 | 30 | 31 | 33 | 35 |
|---|---|---|---|---|---|---|---|---|---|---|
| y | 29 | 18 | 20 | 22 | 27 | 21 | 29 | 27 | 29 | 28 |

Also, calculate the age of husband corresponding to16 years age of wife. {07*Marks*}

## PART – B

5. a) Define i) sample space and ii) event. With usual notation, prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ for any arbitrary events A and B. {06*Marks*}

b) In a bolt factory 25%,35% and 40% of the total is manufactured by machines $M_{1,} M_2$ *and* $M_3$ respectively out of which 5%,4%, 2%,are defective. What is the probability that a bolt drawn at random is defective? What is the probability that it is from $M_{1,}$ ? {07*Marks*}

c) Define random variables and classify. The random variable $X(=x)$ has the following probability distribution for various values of x:

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $p(x)$ | 0 | k | 2k | 2k | 3k | $k^2$ | $2k^2$ | $7k^2 + k$ |

(i) Find k (ii) evaluate p(x<6) , p(x$\geq$ 6), *and* $P(3 < x \leq 6)$ {07*Marks*}

6. a) The probability that a pen manufactured by a factory be defective is 1/10. If 12 such pens are manufactured , what is the probability that (i)exactly 2 are defective (ii) at least 2 are defective (iii) none of them are defective.

{06*Marks*}

b) A shop has 4 disel generator sets which it hires every day. The demand for a generator set on an average is a poison variate with value 5/2. Obtain the probability that on a particular day (i) there was no demand (ii) a demand had to be refused.

{07*Marks*}

c) Define probability density function of the normal distribution. An analog signal received as a defective (measured in micro volts) may be modeled as a normal random variable with mean 200 and variance 256 at a fixed point of time. What is the probability that the signal will exceed 240 micro volts.

{07*Marks*}

7. a) The joint probability distribution table for tow random variables X and Y is as follows.

| X \ Y | -2 | -1 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0.1 | 0.2 | 0.0 | 0.3 |
| 2 | 0.2 | 0.1 | 0.1 | 0.0 |

Compute (i) $E(X)$ *and* $E(Y)$ (ii) $E(XY)$ (iii) $Cov(X,Y)$ $\{06 Marks\}$

Contd…3



b) Define a Markov chain. Prove that the Markov chain with transition matrix $P = \begin{bmatrix} 0 & 1/2 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is irreducible. Find the corresponding stationery probability vector. $\{07 Marks\}$

c) Three boys A, B and C throwing a ball to each other. A always throws the ball to B and B always throws the ball to C. But C is just as likely to throw the ball to B as to C. If C was the first person to throw the ball , find the probabilities that A has the ball (ii) B has the ball and (iii) C has the ball, for the fourth throw. $\{07 Marks\}$

8. a) Define rank of a matrix. Find the rank of the matrix: $\{06 Marks\}$

$$\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

b) Solve the system of equations, by Gauss-Seidel method: $\{07 Marks\}$

$$3x + 4y + 5z = 18$$

$$2x - y + 8z = 13$$

$$5x - 2y + 7z = 20$$

c) Define eigen value and eigen vector of a matrix. Find the dominant eigen value and the corresponding eigen vector of the matrix: $\{07 Marks\}$

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

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## DEPARTMENT OF MATHEMATICS

### FOURTH SEMESTER B.E. – ACADEMIC YEAR: 2009-10

*Course with Code: Engineering Mathematics-IV(P08MAEC41)*

**(For E&C Branch)**

*Model Question Paper*

**Credits : 4-0-0** **Time: 03 Hours** **Max.Marks:100**

**NOTE:- Answer any FIVE full questions choosing at least TWO questions from each part.**

**PART – A**

1. a) Define an analytic function and obtain Cauchy-Riemann equations in the Cartesian form. $\{06 Marks\}$

   b) Determine the analytic function $f(z)$ whose imaginary part is $\left(r - \frac{k^2}{r}\right)\sin\theta,\ r \neq 0$. Hence find the real part of $f(z)$ and prove that it is harmonic. $\{07 Marks\}$

   c) Find the bilinear transformation which maps the points $z = 1,\ i, -1$ into $w = 2, i,\ -2$. Also, find the invariant points of the transformation. $\{07 Marks\}$

2. a) State and prove Cauchy's integral formula. $\{06 Marks\}$

   b) Expand $f(z) = \frac{z+1}{(z+2)(z+3)}$ in terms of Laurent's series valid in the regions $(i)\ |Z| > 3 \qquad (ii)\ 2 < |Z| < 3$ $\{07 Marks\}$

   c) State Cauchy's residue theorem. Using the same evaluate $\int_C \frac{e^{2z}}{(z+1)(z-2)} dz$, where C is the circle $|Z| = 3$. $\{07 Marks\}$

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by the method of least squares. $\{07 Marks\}$ Contd…2



c) The following data gives the age of husband ($x$) and the age of wife ($y$) in years. Calculate the coefficient of correlation and lines of regression

| x | 36 | 23 | 27 | 28 | 28 | 29 | 30 | 31 | 33 | 35 |
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(i) Find k (ii) evaluate p(x<6) , p(x≥ 6), *and* $P(3 < x \leq 6)$ $\{07 Marks\}$

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   c) Define probability density function of the normal distribution. An analog signal received as a defective (measured in micro volts) may be modeled as a normal random variable with mean 200 and variance 256 at a fixed point of time. What is the probability that the signal will exceed 240 micro volts. $\{07 Marks\}$

6. a) The joint probability distribution table for tow random variables X and Y is as follows.

| X \ Y | -2 | -1 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0.1 | 0.2 | 0.0 | 0.3 |
| 2 | 0.2 | 0.1 | 0.1 | 0.0 |

Compute (i) $E(X)$ *and* $E(Y)$ (ii) $E(XY)$ (iii) $Cov(X, Y)$ $\{06 Marks\}$

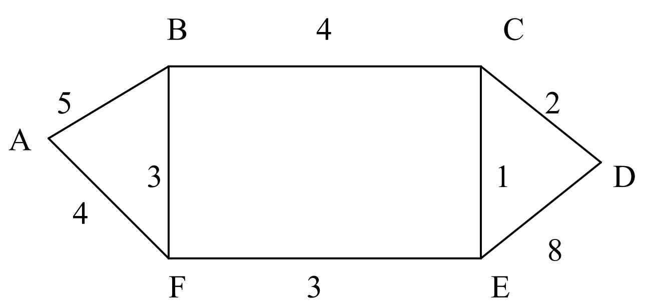
b) Define a Markov chain. Prove that the Markov chain with transition matrix

$$P = \begin{bmatrix} 0 & 1/2 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$ is irreducible. Find the corresponding stationary probability vector. $\{07 Marks\}$

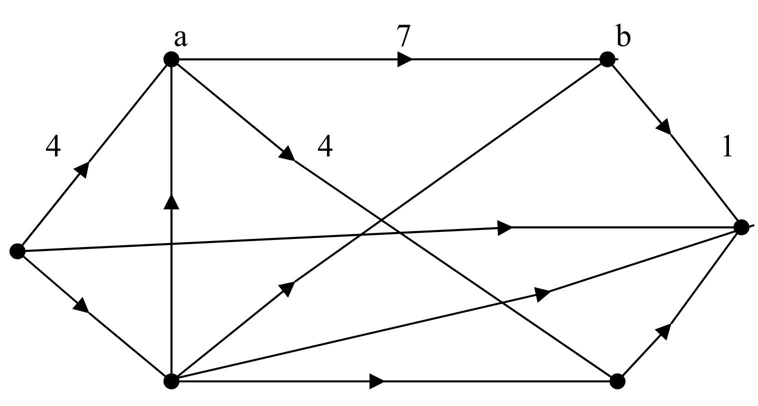
c) Three boys A, B and C throwing a ball to each other. A always throws the ball to B and B always throws the ball to C. But C is just as likely to throw the ball to B as to C. If C was the first person to throw the ball , find the probabilities that (i) A has the ball (ii) B has the ball and (iii) C has the ball, for the fourth throw. $\{07 Marks\}$

7. a) Define planar graphs, digraphs and spanning tree with respect to a connected graph. $\{06 Marks\}$

b) State Kruskal's algorithm. Apply Kruskal's algotithm to find a minimal spanning tree for the weighted graph given below: $\{07 Marks\}$




c) Find the shortest path between K and L in the graph shown below by using Dijkistras algorithm: $\{07 Marks\}$



1 20

K L

5 6

2 2

c 6

8. a) Define rank of a matrix. Find the rank of the matrix: $\{06 Marks\}$

$$\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

b) Solve the system of equations, by Gauss-Seidel method: $\{07 Marks\}$

$$3x + 4y + 5z = 18$$
$$2x - y + 8z = 13$$
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c) Define eigen value and eigen vector of a matrix. Find the dominant eigen value and the corresponding eigen vector of the matrix: $\{07 Marks\}$

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

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**Register Number__________**

# P.E.S. COLLEGE OF ENGINEERING: MANDYA-571 401

(An Autonomous Institution under VTU, Belgaum)

## DEPARTMENT OF MATHEMATICS

### FIRST SEMESTER M.C.A. – ACADEMIC YEAR: 2008-09

### Course with Code: Mathematics (P08MCA101A)

*Model Question Paper*

**Credits: 05(4:1:0)** **Time:03 Hours** **Max.Marks:100**

**NOTE:- Answer any FIVE full questions.**

1. a) If $\sin\theta = \frac{3}{4}$ ($\theta$ is acute angle), find the values of i) $\cos\theta$ ii) $\tan\theta$

   b) With usual notations, prove that cos (A+B) = cosAcosB – sinA sinB

   c) Prove that $(1+\cos\theta+\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\frac{\theta}{2}\cos\left(\frac{n\theta}{2}\right)$

**(6+7+7 = 20)**

2. a) $\cos\alpha+\cos\beta+\cos\gamma = 0 = \sin\alpha+\sin\beta+\sin\gamma$ Prove that

   i) $\cos 3\alpha + \cos 3\beta + \cos\gamma = 3\cos(\alpha+\beta+\gamma)$

   ii) $\sin 3\alpha + \sin 3\beta + \sin\gamma = 3\sin(\alpha+\beta+\gamma)$

   b) Express $Cos^8 x$ in terms of cosine series

   c) Find the cube root of 1 + i and represent them in the Argand diagram.

**(6+7+7 = 20)**

3. a) Define the following with example i) Square matrix ii) Symmetric matrix iii) Orthogonal matrix

   b) If $A = \begin{pmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{pmatrix}$ *and* $C = \begin{pmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & -2 & 3 \end{pmatrix}$

   Verify that A(B+C) = AB + AC.

   c). Find the rank of the matrix $\begin{pmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$

**(6+7+7 = 20)**

**Contd…2**

4.a) Test the consistency and hence solve: 2x + 5y + 7z = 52, 2x + y –z = 0, x + y + z = 9.

b) Find eigen values and eigen vectors of $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

c) Using Cayley – Hamilton theorem, find the inverse of the matrix $\begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$

**(6+7+7 = 20)**

5. a). Differentiate $\log(\sin x) + e^{\sin^{-1} x}$ w.r.t x.

b). If $y = a\cos(\log x) + b\sin(\log x),\ prove\ that x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$

c). Find the n[th] derivative of $i).e^{-x}\cos^2 x$ $ii).\dfrac{x}{(x-1)(2x+3)}$

**(6+7+7 = 20)**

6. a) Evaluate: i) $\lim\limits_{x\to 0}\left(\dfrac{a^x + b^x + c^x + d^x}{4}\right)^{1/x}$ ii) $\lim\limits_{x\to \pi/2}\dfrac{\log(x-\pi/2)}{\tan x}$

b) Show that the pairs of curves $r = a(1+\cos\theta)$ & $r = b(1-\cos\theta)$ intersect orthogonally.

c) Find the pedal equations the curve $r^2 \sin 2\theta = a^2$

**(6+7+7 = 20)**

7. a) Evaluate: $i)\displaystyle\int \frac{1}{(2x+3)^3}dx$ ii) $\displaystyle\int \frac{1}{1+e^x}dx$

b) Evaluate : $\displaystyle\int \frac{1}{5+3\cos x}dx$

c) Evaluate ; $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$

**(6+7+7 = 20)**

8. a) Solve: $\dfrac{dy}{dx} = \cos(x+y+1)$

b). Solve: $r\sin\theta - \cos\theta\left(\dfrac{dr}{d\theta}\right) = r^2$

c). Solve: $(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$

**(6+7+7 = 20)**

4.a) Test the consistency and hence solve: 2x + 5y + 7z = 52, 2x + y –z = 0, x + y + z = 9.

b) Find eigen values and eigen vectors of $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

c) Using Cayley – Hamilton theorem, find the inverse of the matrix $\begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$

**(6+7+7 = 20)**

5. a). Differentiate $\log(\sin x) + e^{\sin^{-1} x}$ w.r.t x.

b). If $y = a\cos(\log x) + b\sin(\log x)$, *prove that* $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$

c). Find the n[th] derivative of $i).e^{-x}\cos^2 x$ $ii).\dfrac{x}{(x-1)(2x+3)}$

**(6+7+7 = 20)**

6. a) Evaluate: i) $\lim_{x\to 0}\left(\dfrac{a^x + b^x + c^x + d^x}{4}\right)^{1/x}$ ii) $\lim_{x\to \pi/2} \dfrac{\log(x - \pi/2)}{\tan x}$

b) Show that the pairs of curves $r = a(1+\cos\theta)$ & $r = b(1-\cos\theta)$ intersect orthogonally.

c) Find the pedal equations the curve $r^2 \sin 2\theta = a^2$

**(6+7+7 = 20)**

7. a) Evaluate: $i)\int \dfrac{1}{(2x+3)^3}dx$ ii) $\int \dfrac{1}{1+e^x}dx$

b) Evaluate : $\int \dfrac{1}{5+3\cos x}dx$

c) Evaluate ; $\int_0^{\frac{\pi}{2}} \dfrac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}dx$

**(6+7+7 = 20)**

8. a) Solve: $\dfrac{dy}{dx} = \cos(x+y+1)$

b). Solve: $r\sin\theta - \cos\theta\left(\dfrac{dr}{d\theta}\right) = r^2$

c). Solve: $(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$

**(6+7+7 = 20)**

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